Abstract

Let $\mathcal{E}$ be a single or a system of equations and let r be a positive integer, $r > 1$. The Rado number, if it exists, is the least positive integer R such that every r-colouring of integers from the interval $[1, R]$ admits a monochromatic solution to $\mathcal{E}$.

We mostly consider the case $r = 2$, and for a variety of classes of equation or equations (i) consider the problem of existence of the Rado number, and (ii) determine upper and lower bounds, and in some cases, exact values of Rado numbers. We also consider Disjunctive Rado numbers. Disjunctive Rado number for a system of two equations E_1, E_2 is the least positive integer D such that every r-colouring of the integers from the intervals $[1, D]$ contains a monochromatic solution to either E_1 or E_2.

We cover Rado numbers, Disjunctive Rado numbers, Schaal equation and its generalization in this thesis. The equations that we worked with are

$$\sum_{i=1}^{m-1} a_i x_i - x_m = c,$$

$$\sum_{i=1}^{m-2} x_i + a x_{m-1} - x_m = c,$$

$$\sum_{i=1}^{m-2} a x_i + b x_{m-1} = b x_m,$$

and

$$a(x_1 + x_2 + x_3) = 3x_4.$$

Our main focus is non-homogeneous equations and how the coefficient set impacts the existence of Rado number and Disjunctive Rado number, along with the constant involved and number of variables.