# Wave Packet Enriched Finite Elements for Multiphysics Wave Propagation Problems in Solids

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## Abstract

The analysis of wave propagation in solids is desired in a number of applications, for example, impact response, characterization of elastic waves for nondestructive evaluation (NDE), guided waves for structural health monitoring (SHM) of thin-walled structures, tremor propagation through ground and thermal shock waves in elastic and piezoelectric components. The finite element method (FEM), which is otherwise a robust numerical tool for solving boundary value problems, has been found wanting in case of different classes of wave propagation problems due to either prohibitively large computational efforts required or errors like numerical dispersion, dissipation and Gibbs phenomenon, introduced in the solution. The various non-conventional finite element methods proposed in the literature suffer from limitations such as the lack of generality of the standard FEM, requirement of prior knowledge of the type of solution, applicability for only a specific class of problems and involving computationally expensive inverse Laplace transform for time domain solution.

In this thesis, a new wave packet enriched finite element (FE) formulation is presented, which is capable of accurately solving a wide range of wave propagation problems in one and two dimensional elastic domains under narrowband and broadband excitations. The Lagrangian interpolation functions of the standard FEM are enriched with the local element domain spatial harmonic functions which satisfy the condition of partition of unity. The nodal displacements are used as the DOFs for the conventional Lagrangian interpolation, while the enrichment functions are made to vanish at the nodes. It allows prescription of boundary conditions in the same way as in the conventional FE method. The Newmark-$\beta$ implicit time integration is employed to solve the equations of motion in time domain. The method is assessed for different classes of wave propagation problems such as the impact and high-frequency guided wave propagation in bars and plates, and surface and body wave propagation in semi-infinite solid media for which the classical FE method either fails to yield accurate results or is prohibitively expensive. It is shown that the present formulation gives accurate solutions to the former and significant improvement

in computational efficiency for the latter category of problems. The performance is also assessed against other special FEs such as the spectral FE and a recently proposed enriched FE with global harmonic basis functions.

Further, a coupled multiphysics enriched FE formulation based on the L–S and G–L generalized piezothermoelasticity theories is presented for accurately solving transient electroelastic, thermoelastic and electrothermoelastic wave propagation problems in 2D elastic and piezoelastic continua. The formulation considers the full thermoelectromechanical coupling consisting of the direct and converse piezoelectric effects, thermoelastic effects and pyroelectric effects and the momentum, charge and energy balance equations. The solution for impact in a piezoelectric plate is shown to alleviate the spurious undulations in both velocity and electric potential fields, which are encountered in the conventional FE solutions. For the problem of high frequency Lamb wave actuation and sensing in a thin plate bonded with piezoelectric actuator/sensor patches, the element shows significant improvement in the computational efficiency over the conventional FE. The free edge effect of steep gradients in the shear stress distribution at the actuator-plate interface is accurately captured by the proposed element using much fewer degrees of freedom than the conventional FE. For the thermoelastic and electrothermoelastic shock wave propagation problems of generalized thermoelasticity/piezothermoelasticity, the method is assessed in comparison with the Laplace transform based analytical solutions and FE solutions with dynamic remeshing available in the literature. The present solution with a static uniform mesh captures the sharp discontinuities in various fields and the wave properties of heat conduction very accurately, practically eliminating the severe drawbacks of the conventional FE solutions. The solution is then used to study the characteristics of wave propagation in thermal shock and the effect of various time constants of the generalized theories on the same.

Finally, the wave packet enriched multifield FE formulation is extended to solve axisymmetric wave propagation problems in elastic and piezoelectric solids and its performance is assessed for the problems of axisymmetric Rayleigh wave propagation in elastic semi-infinite domain and thermal shock propagation in annular elastic disc and hollow piezoelectric cylinder.