Abstract

Synchronization is a fundamental phenomenon in the collective dynamics of coupled oscillators whereby oscillators with different phases, when coupled, adjust their rhythm to a common phase. Examples include synchronization of pacemaker cells in the heart, synchronization of physiological rhythms, cohesive flashing of fireflies, clock synchronization in decentralized computing, frequency synchronization in complex power networks, crowd synchrony on London's Millennium Bridge, and synchronization of neural oscillators for perceptual decision making. The Kuramoto model is the most popular model to investigate the synchronization phenomena in coupled oscillators. Various researchers have found sufficient coupling gains that yield synchronization of Kuramoto oscillators. While synchronization in the one dimensional Kuramoto oscillators has been studied, similar issues for synchronization of higher-order oscillators are relatively unclear. This thesis investigates sufficient coupling gains for the synchronization of higher-order oscillators. We address this issue for benchmark oscillators using a combination of theoretical methods, such as the Lyapunov stability theorem, LaSalle's invariance principle, concepts of non-smooth Lyapunov functions, upper Dini-derivatives, and basic graph-theoretic notions of Laplacian matrix and Spanning trees, as well as numerical simulations and experimental testing. The primary contributions of this thesis are,

- Synchronization analysis has been performed using quadratic Lyapunov function and non-smooth Lyapunov function approaches for the second-order Van der Pol and Fitzhugh Nagumo oscillators. The cases of all-to-all topology and arbitrary connected network topologies with

linear diffusive couplings have been investigated to determine the sufficient coupling gains for synchronization. It is observed that synchronization would occur if the coupling gain between oscillators is sufficiently large. The results have been confirmed through simulations.

• Electronic set-ups for the Van der Pol oscillators and the Fitzhugh Nagumo oscillators have been fabricated and tested for synchronization. The cases of three all-to-all networked oscillators and four-star networked oscillators have been covered for each of the above types of oscillators to investigate the practical threshold of coupling gain synchronization. Possible reasons for discrepancies in analytical and experimental results are discussed.

• Networks of third-order oscillators, namely the Hindmarsh Rose oscillator, the Goodwin oscillator, and the Ring oscillator, have been investigated to achieve synchronization using the non-smooth Lyapunov function approach. The cases of all-to-all and arbitrary connected oscillators' networks have been covered for each of these third-order oscillators. It was found that these oscillators synchronize for coupling gains more than some threshold values. Simulations have confirmed these results. Next, experimental electronic set-up for three all-to-all coupled Hindmarsh Rose oscillators and four all-to-all networked Ring oscillators were developed. Again, it was observed that synchronization occurs in these third-order benchmark oscillators with single state linear diffusive couplings when coupling gain is more than some threshold.

• The pinning control of coupled second-order oscillators has been investigated for independent networks of Van der Pol as well as Fitzhugh Nagumo oscillators with all-to-all as well as arbitrary connected network topologies using quadratic Lyapunov function approach. Simulations confirm the results. Electronic experimental set-ups have been fabricated for these oscillators, and synchronization is practically achieved by using a single pinning controller. Overall, this thesis presents case studies of synchronization of coupled second-order and third-order benchmark oscillators that may be relevant for understanding the consensus phenomena in complex oscillatory networks.