Thesis Title: Graph Coloring and Its Variations: Structural and Algorithmic study

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Abstract

Graph coloring is one of the important and widely studied areas of research in graph theory. A proper k-coloring of a graph G is an assignment of k colors to the vertices of G such that no two adjacent vertices receive the same color. In this thesis, we study the algorithmic aspects of four variations of graph coloring, namely (i) Grundy coloring, (ii) partial Grundy coloring, (iii) total coloring, and (iv) adjacent vertex distinguish (AVD)-total coloring.

A Grundy k-coloring of a graph G is a proper k-coloring such that for every vertex v of color i, v has a neighbor of color j, for all $j < i$. Any Grundy coloring of a graph G is a proper coloring generated by the well known heuristic, *greedy coloring algorithm*, which takes an ordering of the vertices, $(v_1, v_2, \ldots, v_n)$ and assign v_i the least color which is not assigned to any neighbor v_j of v_i, for $j < i$. The Grundy number of a graph G is the maximum number of colors used by the greedy coloring algorithm to properly color G. A partial Grundy coloring is a proper coloring such that for every color i, there exists a vertex of color i having at least one neighbor of color j, for all $j < i$ and the maximum number of colors used for any partial Grundy coloring of a graph G is called the partial Grundy number of G.

In this thesis, we strengthen the existing NP-completeness results for the problems to find the Grundy number and the partial Grundy number for bipartite graphs by giving the NP-completeness results for these problems in some subclasses of bipartite graphs. On the other hand, we propose some polynomial time algorithms to solve these problems in a few subclasses of bipartite graphs. In this thesis, we improve an existing upper bound on the partial Grundy number. We also give an NP-completeness result for the problem to find the partial Grundy number, for doubly chordal graphs and give a linear time algorithm for split graphs. Further, we prove that the problem remains NP-complete for the complement of bipartite graphs.

In the other part of the thesis, we study total coloring and AVD-total coloring in various graph classes. A total coloring is an assignment of colors to the vertices and edges of the graph such that no two adjacent vertices or edges receive the same color and the end vertices of an edge receive different colors. An AVD-total coloring is a total coloring of a given graph G such that for any adjacent vertices u and v, the color set of u and v are different, where the color set of a vertex contains the colors of the edges incident on that vertex and the color of the vertex itself. The minimum number of colors used for a total (AVD-total) coloring of a given graph G is called the total (AVD-total) chromatic number of G. The total chromatic number of a graph G is at least $\Delta(G)+1$, and it is conjectured that it is at most $\Delta(G)+2$, which is known as *total coloring conjecture*.

It is known that the problem to classify the graphs according to their total chromatic number is NP-hard even for 3-regular bipartite graphs. In this thesis, we study the total chromatic number in two subclasses of bipartite graphs, namely, biconvex graphs and chain graphs. In this work, we also consider the graph operations: central graphs and corona products. We prove that the total coloring conjecture holds for the central graph of a graph. We study the AVD-total chromatic number of the central graphs of some basic graph classes and obtain some results on the AVD-total chromatic number of corona products of graphs. We also give some results on AVD-total coloring of split graphs.