TITLE AND ABSTRACT

Title of the thesis. A study on the convolution sums of the divisor functions and related questions

Abstract. One of the most important open problems in number theory is the 'Lindelöf hypothesis', which predicts the growth of the Riemann zeta function on the critical line. One of the earliest attempts to solve this problem was made by studying the moments of the Riemann zeta function. It turns out that in order to estimate the moments, it is enough to understand the convolution sums of the generalised divisor functions.

In fact, convolution sums of arithmetic functions lie at the heart of analytic number theory. In this thesis, we explore some questions related to the convolution sums of the divisor functions. Broadly speaking, we derive explicit lower bounds for the summations appearing in the context of various binary additive problems.

Motivated by the pioneering work of Ingham, we study the additive convolution sub-sum of the divisor function. Moreover, we also study the additive convolution sub-sum of two arithmetic functions having absolutely convergent Ramanujan expansions with certain growth conditions on their Ramanujan coefficients.

Perhaps the most famous appearance of the additive convolution sums is in the context of the Goldbach conjecture. The best known result in the direction of Goldbach conjecture is due to Chen. Applying Selberg's linear sieve, we study Chen's theorem over small primes for odd positive integers, i.e., a variant of Lemoine's conjecture.

We also study the triple convolution sums of the divisor function and the generalised divisor functions. The asymptotic behaviour of the triple convolution sums of the divisor function is not yet known. Browning conjectured a precise asymptotic formula for this. Although the order estimates follow from the classical sieve theoretic results, no explicit bound for the triple convolution sum was known. Invoking the theory of the multiple Dirichlet series, we derive an explicit lower bound for the triple convolution sum of the divisor function.

We extend Browning's conjecture by considering the triple convolution sums of the generalised divisor functions. Finally, we show that our methods can also be applied in studying the convolution sums of the generalised divisor functions over primes.