Numerical Investigation of Wave Turbulence in Nonlinear Acoustic and Fluid Systems

Abstract

Wave turbulence represents a fundamental class of nonlinear phenomena in which energy is redistributed across a broad range of spatial and temporal scales through interacting dispersive waves. Such dynamics arise in diverse physical systems, including fluids, plasmas, nonlinear optical media, and Bose–Einstein condensates. Despite significant progress in weak wave turbulence theory, many realistic systems operate beyond idealized assumptions, where finite-amplitude effects, coherent structures, dissipation, and external driving play a crucial role in the transition from ordered wave motion to fully developed turbulence. This thesis presents a comprehensive numerical investigation of wave turbulence generation, stability, and energy spectrum evolution in fluid systems using nonlinear dispersive wave models derived from fundamental conservation laws.

The work begins with the derivation and numerical analysis of a modified nonlinear Schrödinger (MNLS) equation, obtained by incorporating energy conservation into the standard compressible fluid equations. In this formulation, the source of nonlinearity arises from temperature variations induced by high-amplitude acoustic waves. Numerical simulations reveal that, unlike the classical NLS equation—which exhibits Fermi–Pasta–Ulam (FPU) recurrence and quasi-periodic dynamics—the MNLS system undergoes a breakdown of recurrence due to enhanced nonlinearity, leading to the formation of localized structures and the onset of wave turbulence. A semi-analytical approach is employed to elucidate the physical mechanism governing the evolution, broadening, and steepening of localized wave packets. The resulting turbulence exhibits a power-law energy spectrum close to the Kolmogorov–Zakharov scaling, indicating a nonlinear cascade mediated by wave–wave interactions.

The thesis further extends the investigation to a Zakharov-like (ZL) model, derived from mass and momentum conservation equations, which captures the coupling between high-frequency acoustic waves and low-frequency density fluctuations. The model is shown to reduce to an MNLS equation under quasi-stationary assumptions. Linear stability analysis confirms the presence of modulational (Benjamin–Feir) instability, while numerical simulations demonstrate the formation of cavities, humps, and chaotic structures. The disruption of FPU recurrence and enhanced mode coupling lead to broadband turbulence characterized by steeper energy spectra compared to the NLS case, reflecting stronger nonlinear interactions.

To examine the role of dissipation, a damped coupled wave–density model is developed and analyzed. The inclusion of damping significantly alters the turbulence dynamics by suppressing coherent recurrence, modifying instability growth rates, and regulating energy transfer across scales. Numerical results show that damping steepens the energy spectra and promotes statistically steady turbulence states consistent with weak wave turbulence predictions. The interplay between density and temperature fields is examined in detail, highlighting their combined influence on turbulence evolution and spectral behavior.

Finally, the thesis investigates a forced and damped Zakharov-type system, focusing on how external driving and frequency detuning control turbulence regimes. Three distinct regimes cavity-dominated, mixed, and cascade-dominated turbulence are identified based on detuning strength. Ensemble-averaged

spectra reveal clear regime-dependent scaling laws, demonstrating how controlled forcing and dissipation shape nonlinear energy cascades.

Overall, this thesis establishes a unified numerical framework for understanding wave turbulence in fluid systems by systematically incorporating energy conservation, nonlinear coupling, damping, and driving into reduced wave models. The results provide new physical insights into the mechanisms governing instability, recurrence breakdown, and spectral energy transfer, bridging the gap between idealized wave turbulence theory and realistic fluid-wave dynamics. The findings are relevant to a wide range of applications, including acoustic turbulence, ocean and atmospheric waves, and other dispersive nonlinear systems.