COMPUTATION OF OPTIMAL CONSENSUS IN MULTI-AGENT SYSTEMS USING GRÖBNER BASIS

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Abstract

This thesis presents a distributed framework for computing optimal consensus in multi-agent systems subject to resource constraints, specifically fuel and energy. We consider a group of N identical linear time-invariant agents with n states, bounded control inputs, and formulate the consensus problem as finding the least time when the attainable sets of all agents intersect, and the corresponding consensus state is the point of intersection. This set-theoretic formulation utilizes the geometric properties of attainable sets and naturally leads to a characterization of fundamental limitations on consensus in terms of the agents' initial conditions, available time, and control resources such as fuel and energy. However, as the number of agents increases, direct computation of the global intersection becomes computationally intractable. To address this challenge, Helly's theorem from convex geometry is used to decompose the computation into $\binom{N}{n+1}$ subproblems. In each subproblem, the minimum time and the corresponding point at which the intersection of attainable sets for a collection of $(n+1)$ agents becomes non-empty are determined. We develop a systematic and distributed procedure to solve these subproblems and to combine their solutions, to find the minimum time to consensus, at which N agents can reach the corresponding consensus point while respecting the bounded energy constraints. The solution of each subproblem reduces to solving a finite set of polynomial equations, for which Gröbner basis–based elimination method results in closed-form expressions and a computationally tractable characterization of the optimal consensus solution. The proposed framework is developed for agents with second-order dynamics, including double integrators and a class of damped second-order linear time-invariant systems under fuel budget constraints. Further, under the energy budget constraints (as opposed to fuel budget), we are able to generalize the framework for general n^{th} order agent dynamics. In particular, we focus on a) minimum time consensus with fuel budget constraints, b) minimum fuel budget allocations for consensus at a given time, and c) minimum time consensus with energy budget constraints.