

PhD Thesis, titled
**GROWTH-INDUCED INSTABILITY AND MORPHOLOGICAL
CHANGES IN HIGHLY FLEXIBLE SLENDER STRUCTURES**

Submitted by
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ABSTRACT

Slender structures such as rods and plates are ubiquitous in both natural and engineered systems. Their highly flexible nature makes them prone to large deformations, geometric nonlinearities, instabilities, and post-buckling responses, which play a crucial role in determining their mechanical behavior. These phenomena are central to a wide range of applications, including soft robotics, flexible electronics, and biological growth processes. However, traditional elasticity theories for modeling such structures often fall short in describing such systems, as they are typically restricted to small deformations and cannot adequately capture the geometric nonlinearities, finite rotations, and intrinsic growth effects that govern their behavior. To overcome these limitations, generalized continuum theories—most notably the Cosserat rod and plate models—have been developed. These models provide a versatile framework that naturally incorporates shear, finite rotations, and complex boundary interactions, allowing for an accurate description of three-dimensional kinematics using one- or two-dimensional continua. Building upon these foundations, recent advances in morphoelasticity have further extended the theoretical scope to include growth and active material responses, thereby enabling the study of growth-driven mechanics in biological and bioinspired systems. This thesis aims to develop non-linear Cosserat-based models for highly flexible slender structures, with particular emphasis on growth-induced deformations.

The first part of this thesis develops a nonlinear constitutive model to determine the natural shapes and stiffnesses of morphoelastic rod-like structures modeled as special Cosserat rods. The formulation allows the rod's cross-section to have an arbitrary shape, and the material to undergo both volumetric and surface growth. The helical Cauchy–Born rule is employed to generate a helical rod in which the strain parameters are imposed uniformly along its arc length. This uniformity in the strain field enables the reduction of the three-dimensional nonlinear equations of morphoelasticity from the entire rod body to its cross-sectional plane. The governing equations for the deformation of the rod's cross-section are then derived by considering both volume and surface growth. Semi-analytical expressions for the stress resultants and stiffnesses of the rod are subsequently obtained in terms of the prescribed strains, growth parameters, and the unknown deformed cross-section. By setting the stress resultants to zero, the natural shape of the rod is

determined. A finite element formulation is also presented to solve the non-linear cross-sectional morphoelasticity problem. To elucidate the presented computational framework, several numerical simulations are performed involving volumetric uniform and differential axial growth, isotropic surface growth and active filaments.

The second part of this thesis develops a one-dimensional constitutive theory for ribbons or strip-like structures, modeled as special Cosserat rods through a reduced formulation derived from a general Cosserat plate theory. Starting with the description of a strip as a general Cosserat plate, the strip is subjected to a strain field that is uniform along its length. The helical Cauchy–Born rule is employed to impose this uniform strain field, deforming the strip into a six-parameter family of helical configurations, where the six parameters correspond to the six strain measures of rod theory. Two vector variables are introduced: one to describe the position of the deformed centerline of the strip’s cross-section, and the other to characterize the orientation of the thickness lines along the strip’s width. The minimization of the strip’s plate energy under the constraint of the aforementioned uniformity in strain field reduces the partial differential equations of plate theory from the entire mid-plane of the strip to just a system of nonlinear ordinary differential equations along the strip’s width line for the above mentioned two vector variables. A nonlinear finite element formulation is further presented to solve this system of equations. This formulation yields the strip’s stored energy per unit length, along with the induced internal forces, moments, and stiffnesses for any prescribed set of six strain measures from rod theory. The proposed scheme is applied to study uniform bending, twisting, and shearing of strips. In the cases of uniform twisting and shearing, the strip is observed to buckle along its width into more complex configurations, which are accurately captured by the present formulation. The results demonstrate that the proposed scheme is more general and accurate than the existing approaches available in the literature.

Finally, the third part of this thesis presents a comprehensive theoretical and numerical framework for modeling and analyzing the growth-induced deformation and instability behavior of a coupled plate–rod system. The governing equations and boundary conditions of a special Cosserat circular plate bounded by a special Cosserat rod are systematically derived using variational principles from the total energy of the coupled system. A finite element formulation is also presented to solve the non-linear equilibrium equations of a growing system. Symmetry-reduced boundary conditions are introduced to efficiently capture post-buckling configurations. Analytical formulations are then presented to determine the critical growth parameter corresponding to both axisymmetric and planar non-axisymmetric deformation modes. Special cases, including Kirchhoff-type plate and rod combinations, are examined to provide simplified closed-form expressions for the buckling thresholds. The study further explores the post-buckling response of the coupled system through numerical simulations, revealing transitions from axisymmetric to planar multi-lobed morphologies. Finally, comparisons among various coupled models, ranging from Cosserat to von Kármán formulations, highlight the influence of kinematic and constitutive assumptions, as well as plate and rod stiffness, on the onset and evolution of buckling instabilities under growth.