Abstract

This thesis deals with the study of the system of translates of a fixed element in three different types of spaces: the Hardy space $H^1(\mathbb{R})$, Orlicz spaces $L^\Phi(\mathbb{R}^d)$ and the Schatten p-classes of operators $\mathcal{T}^p$.

The first is the Hardy space $H^1(\mathbb{R})$. First, we characterize those functions whose all translates are complete in terms of the zero set of their Fourier transform. Then we consider the completeness of systems generated by discrete translates. We characterize all the discrete sets $\Lambda \subset \mathbb{R}$ in $H^1(\mathbb{R})$ for which there exists a function whose Λ-translates are complete. These sets are precisely those whose Beurling-Malliavin density is infinite. As a result, uniformly discrete translates of a single function can never be complete. However, we show that the situation is different if we take system of translates of two functions. By taking Λ to be a very small perturbation of integers, there exists a pair of functions such that their Λ-translates are complete in $H^1(\mathbb{R})$.

The second are the Orlicz spaces $L^\Phi(\mathbb{R})$. We study the completeness and Schauder frames properties of system of translates in these spaces based on the properties of the corresponding Orlicz function Φ. For Orlicz functions spaces satisfying $\Phi \in \Delta_2$ with $\lim\limits_{x\to\infty} \frac{\Phi(x)}{x} > 0$, we prove an analogue of the Wiener-Tauberian theorem characterizing those functions in the space $L^\Phi(\mathbb{R})$ whose all translates. For Orlicz functions spaces satisfying $\Phi \in \Delta_2$ with $\lim\limits_{x\to\infty} \frac{\Phi(x)}{x} = 0$, we prove the completeness of all translates of a simple step function in the corresponding Orlicz space. Next we prove an inclusion result on the completeness of translates for the Orlicz functions in the Δ_2 class. We show that a particular partial order of the Orlicz functions, the existence of a function in one Orlicz space whose Λ-translates are complete will imply the same in another. Lastly, we prove the existence of Schauder frames of translates for Orlicz spaces in the Δ' class by comparing them with x^2 under a different partial order. We also show that by imposing auxiliary conditions on Φ, we can obtain additional properties in the frame structure such as unconditionality and the existence of a subsequence with semi-normalized coordinates which is also an unconditional Schauder frame.

The third are the Schatten p-class $\mathcal{T}^p$ which are operator spaces on the Hilbert space $L^2(\mathbb{R}^d)$. We study the completeness and frame properties of systems of the form $\{\alpha_\lambda S\}_{\lambda\in\Lambda}$ in the space $\mathcal{T}^p$ where $\alpha_\lambda S$ denote the translation of the operator S by λ and $\Lambda \subseteq \mathbb{R}^{2d}$ is discrete. We consider the completeness of a system of discrete translates in $\mathcal{T}^p$ for all $p > 1$. We give an example of a uniformly discrete $\Lambda \subset \mathbb{R}^{2d}$ such that there exists an operator whose Λ-translates are complete in $\mathcal{T}^p$ for all $p > 1$. Then we study Schauder frames of translates in $\mathcal{T}^p$. For all $p > 2$, we established the existence of operators whose integer translates give rise to unconditional Schauder frames in $\mathcal{T}^p$.