We study the classical problems of multicommodity flow and cuts and prove new (approximate) max-flow min-cut theorems. In multicommodity flow problems, we are typically given an edge capacitated graph (called the supply graph) and several source-sink pairs (called the demand edges). The objective is to route flow between the source-sink pairs without violating the edge capacities. A flow is integral if each path carries an integer flow. When all the edge capacities are unit, an integral flow corresponds to the well-known edge-disjoint paths problem.

In sum-multicommodity flow, we seek to find a flow that maximizes the total flow routed between the source-sink pairs. A set of edges whose removal disconnects all the source-sink pairs is called a multicut. It follows from the definition that the value of any multicut is at least the value of the maximum sum-multicommodity flow. We show that when the union of the supply and the demand graph is planar, the value of the minimum multicut is at most four times the maximum integral sum-multicommodity flow. Our proof of the above fact is algorithmic, which gives an approximation algorithm for the maximum edge disjoint paths problem and minimum multicut in such settings as well. We make interesting connections to connectivity augmentation problems and the four-colour theorem to prove our results.

In the demand-multicommodity flow problem, we are given a demand value for each source-sink pair and the objective is to find a feasible routing of all the demands if possible. A necessary condition for routing the demands, known as the cut-condition, is as follows: the total capacity of the supply edges across any cut should be at least the total value of the demand edges across it. In general, the cut-condition is not sufficient for existence of a feasible routing. The minimum factor by which the cut-condition must be relaxed to ensure the existence of (integral) flow is called the (integral) flow-cut gap of the instance. We make progress towards proving tight integral flow-cut gap results for series-parallel graphs by exploring interesting connections with matchings in general graphs. We also prove new flow-cut gap results for settings which generalize the classical theorems of Okamura and Seymour, ie. every source-sink pair lies on one of the faces of the planar graph.