Abstract

Interface problems have always been the most challenging research area because of the low global regularity of the solution caused by the discontinuity and the high curvature of the interface exhibited by the interface's shape. The classical analysis of singularly perturbed interface problems is not yet developed and challenging to implement. The analysis and numerical solution of interface problems deservedly attract substantial attention. This thesis design and analyze numerical techniques for some singularly perturbed interface problems.

A two-dimensional singularly perturbed convection-reaction-diffusion problem that has discontinuities, along lines parallel to x- and y-axes, in the source term, as well as in the convection and reaction coefficients is considered. The coefficient of the highest-order term is a small positive parameter. We propose a decomposition of the solution that yields sharp bounds on its derivatives. A particular finite difference scheme is constructed on an appropriate Shishkin mesh, and it is established that the computed solution is almost first-order parameter-uniformly convergent. For the same problem, a local discontinuous Galerkin (LDG) method is constructed on an appropriate Shishkin mesh. The test functions in the LDG method are piecewise polynomials that lie in the space $\mathscr{Q}_r$ of piecewise polynomials of degree at most r in each variable, where r is a positive integer. We established that the error in the computed solution converges at the rate of $r+\frac{1}{2}$ in a DG-norm.

We next design and analyze a parameters-uniform numerical method for a time-dependent weakly coupled system of two convection-diffusion equations that has a discontinuity, along the line $x = d$, in the source term. The second-order term of each equation is multiplied by a small singular perturbation parameter of different magnitude. The convergence analysis of the numerical method is given using the decomposition of the solution into regular and singular components. The numerical approximations produced by the numerical method are uniformly convergent of first-order in time and almost first-order in space with respect to both perturbation parameters.

A finite difference method for a system of $m(\geq 2)$ singularly perturbed parabolic semilinear reaction-diffusion equations with a discontinuous source term having discontinuities along $\Gamma_d = \{(x,t) : \; x = d, \; 0 < t \leq \mathsf{T}\}$. The semilinear operator is linearized, and a maximum principle is proved for the linearized operator. A Green's function is introduced for the semilinear operator using the linearized operator. Certain bounds on the Green's function and its derivatives are obtained. Using these bounds, we give computable parameter-uniform error estimates in maximum norm.

Next, we construct and analyze a numerical method for a time-dependent weakly coupled system of two singularly perturbed semilinear reaction-diffusion equations. The source terms in both equations have discontinuities in the spatial variables along $x = d, \; d \in \Omega := (0,1)$. The highest order spatial derivatives in the first and second equations are multiplied by positive perturbation parameters ε_1 and ε_2, respectively, which could be arbitrarily small. The domain is discretized using an appropriate Shishkin mesh. Using the decomposition of the solution, we obtain $(\varepsilon_1, \varepsilon_2)$-uniform error estimates in "maximum norm." We prove that the method is a $(\varepsilon_1, \varepsilon_2)$-uniformly convergent of first-order in time and almost second-order in space concerning perturbation parameters.

Numerical experiments are conducted on some test problems for all the developed numerical methods to validate the theoretical results.