STUDIES ON EFFICIENT ALGORITHMS FOR CONVEX AND NON-CONVEX PROBLEMS WITH APPLICATIONS IN SIGNAL PROCESSING, COMMUNICATION, AND MACHINE LEARNING

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Abstract

The efficiency of an optimization algorithm plays a crucial role in the performance of applications in signal processing, communication, and machine learning. To tackle the associated optimization problem, several generic optimization algorithms have been proposed. However, a common issue with these optimization algorithms is that they are restrictive and cannot be applied to solve optimization problems of all flavors. Moreover, their performance degrades when the underlying problem takes a complicated structure. To handle these issues, in this thesis, the focus is on utilizing the Majorization Minimization (MM) framework, which is flexible in solving a broad spectrum of problems. For instance, apart from solving convex problems, it can solve problems whose cost functions are non-convex and non-smooth. Also, it can handle problems with non-convex constraints. Importantly, MM also aids in exploiting the problem structure to develop efficient iterative optimization algorithms. Given the various benefits of MM, in this thesis, the MM framework is explored to solve convex, non-convex, and non-smooth problems that have applications in the field of signal processing, communication, and machine learning. Also, the possibility of collaborating the MM framework with generic optimization algorithms is studied.

In the first part of the thesis, the focus is on developing efficient optimization algorithms for the problems encountered in the field of signal processing. In this regard, four different problems related to signal processing applications are considered - 1) Source localization 2) Non-negative matrix factorization 3) Atomic Norm Minimization and, 4) Toeplitz covariance matrix estimation problems. For the first problem, which is a non-convex and non-smooth problem, an optimization algorithm named SOLVIT is proposed. SOLVIT is an MM-based iterative algorithm which, unlike the state-of-the-art algorithms, tackles the problem exactly without any approximations. Next, the non-negative matrix factorization problem, which is a non-convex problem with multiple optimization variables, is studied. By inspecting the structure of this problem, two different iterative optimization algorithms named INOM and PARINOM are presented. While the former optimization algorithm is based on the block MM framework - an extension of MM technique employed to solve problems with several block variables, the latter optimization algorithm is based on the principle of MM procedure. Subsequently, for the convex Atomic Norm Minimization and the non-convex Toeplitz covariance matrix estimation problems, iterative optimization algorithms named DYANOM and ATOM are proposed. To develop both the optimization algorithms, a hidden structure of the underlying optimization problem is exploited. Moreover, DYANOM and ATOM is based on the integration of the MM framework with generic optimization algorithms. For each problem, numerical simulations were conducted, which indicated that the proposed optimization algorithms had a superior performance when compared to the state-of-the-art algorithms.

The second part of the thesis deals with tackling two different optimization problems that have applications in the field of communication. The first problem is concerned with the construction of unimodular sequences with good autocorrelation properties. To attain such sequences, an iterative optimization algorithm named SLOPE is proposed that minimizes the maximum peak sidelobe level of a unimodular sequence. Therefore, SLOPE optimizes a non-convex minimax problem. SLOPE arrives at a stationary point of the problem by leveraging the adaptation of MM framework for minimax problem coupled with the utilization of a generic algorithm. Later, SLOPE is also extended to handle additional constraints such as the energy, peak-to-average-power ratio, and spectral constraints. Numerical simulations confirm that SLOPE can generate sequences of considerably longer lengths with lower peak sidelobe levels. Next, a different non-convex minimax optimization problem is considered. This problem is

used to find optimal frames, which comprises of a set of unit norm vectors such that the maximum correlation among them is minimal. To solve this problem, two iterative optimization algorithms named TELET and FLIP with different updating schemes are proposed. While the former MM-based algorithm, updates all the columns comprising the frame vectors at once, the latter block MM-based algorithm updates the frame in a column-wise fashion. In addition to utilizing MM framework, similar to SLOPE, even TELET and FLIP borrows generic optimization algorithm to construct optimal frames. Numerical simulations reveal that TELET and FLIP can construct frames of longer lengths with lower coherence values.

In the third part of the thesis, the focus is on solving optimizations problems that have applications in the field of machine learning. Under this field, the first problem is concerned with the maximum likelihood parameter estimation for the multinomial logistic regression classifier. Although the problem is convex and differentiable, it does not enjoy a closed-form solution. To solve this problem, a MM-based iterative algorithm named PIANO is proposed which can parallely update each element of the optimization variable. Moreover, unlike the state-of-the-art algorithms, PIANO can also be easily modified to handle the parameter estimation problem for the sparse multinomial logistic regression classifier. Simulations were conducted to compare PIANO with the existing methods, and it was found that PIANO performs better than the existing methods in terms of speed of convergence. Then, a convex ℓ_p norm linear regression problem is considered. To tackle this problem, an iterative optimization algorithm named PROMPT is proposed based on the MM technique. Unlike the state-of-the-art algorithms, PROMPT can handle the underlying cost function for any value of p. Moreover, PROMPT updates each element of the optimization variable parallely. Numerical simulations highlight that PROMPT performs better than the state-of-the-art algorithms in terms of speed of convergence.