ABSTRACT

In this thesis, our focus is to study certain properties of iterates of polynomials over a field. In fact, we discuss the irreducibility and monogenity of iterates of a polynomial. To study the irreducibility of polynomials over a field is important due to its applications in number theory and in other areas of mathematics. We investigate the irreducibility of iterates of the binomial $x^d + \frac{1}{c}$ for a nonzero integer c and $d \geq 3$, and prove that for an infinite family of these irreducible binomials, all their iterates are also irreducible over $\mathbb{Q}$. Next, we give a conditional result and show that if the binomial $f(x) = x^d + \frac{1}{c}$ is irreducible, then all its iterates are also irreducible under the well-known abc-conjecture. Moreover, when $d = 3$, we show that the number of irreducible factors of iterates of $f(x)$ is atmost 2 if $|c| \leq 10^{12}$.

We also study the irreducibility of iterates of certain trinomials over finite fields of odd characteristics. Furthermore, we prove that for a family of trinomials of prime degree p, its second iterate is a product of p irreducible polynomials, each of degree p. Besides, we give a criterion for a family of polynomials of certain type to be "dynamically irreducible" over a finite field and produce dynamically irreducible sets of polynomials.

One of the oldest problems in algebraic number theory is finding the ring of integers in a number field. If the ring of integers of a number field K is $\mathbb{Z}[\theta]$ for some $\theta \in K$, then K is referred to as a monogenic field and the minimal polynomial of θ over $\mathbb{Q}$ is called a monogenic polynomial. We also study the monogenity of the iterates of certain families of polynomials. To discuss the monogenity of the iterates of a polynomial, we need the iterates of the polynomial to be irreducible. First, we give a necessary condition for the iterates of a polynomial to be monogenic. If a polynomial satisfies certain Eisenstein criteria, we give a sufficient condition for the iterates to be monogenic. In fact, using this result, we give a couple of examples of polynomials that give an infinite tower of monogenic number fields. Finally, we give a condition which is both necessary and sufficient for the monogenity of the iterates of an irreducible binomial.