The thesis performs a detailed investigation into the dynamics of gas-filled bubbles, motivated by applications such as biomedical ultrasound and cavitation. By combining symbolic computation, asymptotic analysis, and numerical validation, this work offers new insights into the transient and steady-state behaviour of a bubble filled with a polytropically expanding non-condensible gas, accounting for or neglecting real-gas volume exclusion effects. The analysis spans a range of physical regimes—including free and forced oscillations, and undamped, viscously damped, and viscoelastically damped conditions.

A major methodological contribution is the development of long-time accurate singular perturbation techniques for bubble dynamics, *e.g.* Poincaré–Lindstedt procedure and multiscale methods and their symbolic extension through computer algebra, thus revealing additional subtleties in the nonlinear response, while also obtaining superior numerical accuracy without compromising analytical tractability. Previous analytical contributions based on regular perturbations cannot uncover the delicate nonlinear interplays in bubble dynamics, as their continuation to higher orders engenders unphysical and unbounded contributions.

Investigation into finite-amplitude effects in free oscillations reveals that the oscillation frequency decreases with amplitude, deviating from the classical Minnaert prediction. The numerically validated analysis also leads to a hitherto unreported finding involving broken symmetries—an initially compressed bubble oscillates at a frequency different from the initially expanded bubble, even if the radial oscillations are initiated by the same degree of stretch/shrink. In departure from linear theory, the bubble is predicted to oscillate around a mean radius larger than its radius at static equilibrium. The amplitude-dependence of oscillation frequency is additionally modulated by surface tension and real gas effects.

To study the damped autonomous dynamics of bubbles in a Newtonian liquid, a two-time-scale-based singular perturbation process was employed to obtain higher-order asymptotic approximations which remain uniformly valid in progressively expanding time domains. The analysis reveals numerically verifiable findings such as narrower trough regions and wider peak regions in the radius-time curve and faster bounce-back from troughs than from peaks. Further, a proposed symbolic extension of the traditional mass-spring-damper-based regular perturbation approach was found to have its strength in accurately modelling overdamped systems but not underdamped ones. The predicted radius-time curves are in reasonable agreement with experimental data on viscously damped oscillations of spark-generated bubbles available from the literature.

In the forced oscillation regime, the thesis constructs and validates an analytical framework based on three time scales that predicts distinct hallmarks of nonlinearity in the frequency response near resonance, such as jump phenomena, bistability, unstable branches, and hysteresis, arising through bifurcations when ultrasound forcing and damping parameters are tuned across certain thresholds. Closed-form expressions for these thresholds are made available for the first time. The analysis is augmented by a vivid graphical interpretation of the onset of nonlinearities, developed through the definition of new curves and loci.

Motivated by biomedical applications where soft tissues are an arena for the dynamics of endogenous and exogenous bubbles, forced bubble oscillations are also modelled in viscoelastic media. The viscoelastic behaviour of a predominantly solid-like medium outside the bubble is modelled using the Standard Linear Solid (Zener) model, where resonant softening and frequency shifts are characterised in terms of viscoelastic parameters embodying stress relaxation effects and reversible storage of elastic energy. The analytical findings are consistently benchmarked against numerical simulations and, where applicable, experimental data, since the available literature tends to pursue these approaches in isolation. The findings of this thesis are relevant to practical scenarios where the bubble dynamics is oscillatory but not small-amplitude, such as cavitation, biomedical diagnostics and therapeutics using ultrasound and/or cavitating or exogenous bubbles, acoustic sensing of natural gas seepage from underwater reservoirs, sonoluminescence, and sonochemistry.