Thesis title: Differential Flatness of Open and Closed-loop Kinematic Chains

Abstract

Under-actuated mechanical systems come with the notable benefit of cost reduction compared to their fully actuated counterparts due to fewer actuators, and often result in energy efficient designs. The drawback of such systems is that planning and controlling trajectories requires advanced formulation. Flatness-based approach however facilitates implementation of trajectory planning and tracking. Planning is reduced to an algebraic enumeration and a feedback controller to track the flat output trajectories can be designed readily.

Having said all these, proving flatness and finding flat outputs for a system is not trivial. Till date, only necessary or sufficient conditions are available to test flatness. Flatness being intimately tied to the system structure requires one to exploit the physical arguments in the search of flat outputs. The fundamental question that we try to answer in this work is: what mechanical elements in a kinematic chain can be altered or added in which manner so that the resultant system is amenable to being under-actuated by being differentially flat. These modifications range from system mass-inertia parameters to added counter-balancing masses or adding passive mechanical elements like springs and dampers.

This work reports elementary systems consisting of single rigid bodies in 2-D space and go on to show that the flatness property of complex kinematic chains can often be inferred from recursive examination of flatness of its smaller sub-units. In this process, we have a new understanding of the flatness demands of mechanical systems which are modeled as connected rigid bodies. Most mechanical systems are shown to be either configuration flat or at the most depend on the first derivative of the variables (velocity). We have proposed new control affine systems wherein torques and forces enter the set of flat outputs.

Closed chain mechanisms are suitable candidates for high speed applications which require low inertia. In this thesis, we explore classes of binary closed-chain differentially flat systems and propose a systematic approach to flattening them. We demonstrate that by carefully choosing mass distribution and adding passive mechanical elements like springs, an under-actuated closed-loop system can be rendered differentially flat. Numerical simulation of trajectory planning for such systems demonstrates the effectivness of the strategy.

Most interestingly, differential flatness is capable of providing feasible trajectories in systems which are non flat, but allow a flat approximation. We introduce a weaker form of differential flatness for mechanical systems which has been termed as 'partial differential flatness' in some literature. Mostly, systems with cyclic or ignorable coordinates exhibit such a property. Under this category, differential flatness of floating chain and snake-like robots is also established.