PH.D. VIVA-VOCE

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Abstract: We investigate the structure of the Rees algebra $\mathcal{R}(I)$ of height two perfect ideals I in a polynomial ring $R = \Bbbk[x_1, \ldots, x_d]$ over an algebraically closed field $\Bbbk$. While defining equations of $\mathcal{R}(I)$ are well understood when I satisfies the condition $(\mathrm{G_d})$, the cases $(\mathrm{G_{d-1}})$ and $(\mathrm{G_{d-2}})$ have remained largely open. The present work fills the gap by providing explicit and computable descriptions of the defining ideal of $\mathcal{R}(I)$ in these weaker settings, and shows that $\mathcal{R}(I)$ is not necessarily Cohen-Macaulay only under stronger conditions.

The study is divided into two main cases. First, we consider almost linearly presented ideals $I = (f_0, \ldots, f_d)$, that is, ideals whose syzygy matrix has linear entries except possibly in the last column, which consists of homogeneous quadratics. Assuming I satisfies $(\mathrm{G_{d-1}})$ bu not $(\mathrm{G_d})$, we derive explicit generators for the defining ideal of $\mathcal{R}(I)$.

In the second case, we treat linearly presented ideals $I = (f_0, \ldots, f_d)$ that satisfy $(\mathrm{G_{d-2}})$ but fail both $(\mathrm{G_d})$ and $(\mathrm{G_{d-1}})$. Again, explicit equations for $\mathcal{R}(I)$ are obtained, accompanied by counterexamples showing that Cohen-Macaulayness need not hold.

These results yield new insight into the arithmetic and geometric properties of the Rees algebra associated with height two perfect ideals beyond the classical $(\mathrm{G_d})$ setting, and provide concrete tools for their computation.