CONVERGENCE ANALYSIS OF REGULARIZED LEARNING ALGORITHMS FOR FUNCTIONAL DATA

This thesis studies the convergence analysis of estimators in the Functional Linear Regression (FLR) and polynomial regression models, conducting a comprehensive examination of regularized learning algorithms within the framework of Functional Data Analysis (FDA). The study is motivated by the growing availability and importance of functional data in many scientific and applied fields, such as engineering, medicine, environmental studies, and finance, where observations are inherently represented as curves or functions across an arc. As the use of these data types increases, there is a growing need for learning algorithms that not only scale efficiently to high-dimensional functional inputs but also come with rigorous theoretical guarantees to ensure their reliability. The main contributions of the thesis are presented as following:

We generalize classical regularization techniques—such as Tikhonov regularization—by utilizing a general spectral regularization method in the framework of reproducing kernel Hilbert spaces. We derive minimax optimal convergence rates for both estimation and prediction errors under commutative and non-commutative settings, significantly extending prior results by considering wider range of Hölder type smoothness conditions on the unknown slope function. As kernel methods are burdened with hight computational cost, we address this issue by exploring two computationally efficient algorithms in the reproducing kernel Hilbert space framework: the kernel Conjugate Gradient (CG) method and the Nyström subsampling method. The kernel CG method utilizes iterative optimization with early stopping rules to mitigate overfitting and reduce computational costs while maintaining optimal convergence rates. The Nyström method, on the other hand, applies low-rank kernel matrix approximations to significantly lower memory and runtime demands. We establish the optimal convergence rates for each algorithm, demonstrating their statistical efficiency alongside their computational benefits.

Further, we extends the analysis of regularized learning algorithms to settings governed by general source conditions, moving beyond conventional assumptions such as operator monotonicity or Lipschitz continuity of the index functions. Moreover, the analysis is carried out in a general framework aimed at learning a vector in general separable Hilbert space.

Finally, we shift our focus on function-on-function polynomial regression models, which generalize the classical FLR framework by incorporating nonlinear interactions between functional predictors and responses. This extension enables the handling of more complex relationships that cannot be captured by linear models alone. In contrast to the kernel-based methods employed in earlier chapters, this chapter adopts a different framework by working directly in the $\mathbb{L}^2$ space. Theoretical analysis establishes convergence rates under general smoothness conditions. In addition, minimax lower bounds are derived, confirming the optimality of the proposed rates.

The contributions of this thesis collectively establish a thorough foundation for regularized functional regression. By addressing both statistical optimality and algorithmic efficiency, the work enhances the practical viability of functional learning methods while deepening their theoretical understanding.