Isotropy Groups of Non-Simple Derivations of Polynomial Algebras

Himanshu

Abstract

Let K denote a field of characteristic zero containing $\mathbb{Q}$, $K^* = K \setminus \{0\}$, and $K[X] = K[x_1, \dots, x_n]$ be the polynomial algebra in n variables over K. This thesis focuses on finding the isotropy groups of non-simple derivations of $K[X]$.

We started by reviewing some literature and known results on K-derivations, which play a crucial role in the work done in this thesis. In Chapter 2, we study the isotropy groups of non-simple derivations of the form $x_1^m x_2^u D$, where $m, u \geq 0$, and D is a simple derivation of $K[x_1, x_2]$. For $m, u \geq 1$, we have proved that the isotropy group of $x_1^m x_2^u D$ is isomorphic to the product of two finite cyclic groups. Further, we have shown that isotropy groups of derivations $x_1^m D$ and $x_2^u D$ are finite, where m, u are positive integers.

In Chapter 3, we study the isotropy group of the Lotka-Volterra derivation of $K[X]$, i.e., a derivation d of the form $d(x_i) = x_i(x_{i-1} - C_i x_{i+1})$, where $C_i \in K$ and $1 \leq i \leq n$, with $x_0 = x_n$ and $x_{n+1} = x_1$. If $n = 3$ or $n \geq 5$, we have shown that the isotropy group of d is finite. However, for $n = 4$, it is observed that the isotropy group of d need not be finite. Indeed, for $C_i = -1$, we observed an infinite collection of automorphisms in the isotropy group of d. Moreover, for $n \geq 3$, and $C_i = 1$, we have shown that the isotropy group of d is isomorphic to the dihedral group of order $2n$. Furthermore, if $C_i = 0$ for all i, the isotropy group of d is isomorphic to a cyclic group of order n.

In Chapter 4, we study the isotropy group of the Jouanolou derivation, a well-known monomial derivation of $K[X]$ of the form

$$d(x_1) = x_2^s, \quad d(x_2) = x_3^s, \quad \dots, \quad d(x_{n-1}) = x_n^s, \quad d(x_n) = x_1^s,$$

where $s \geq 1$. For $s \geq 2$, we show that its isotropy group is generated by the sets Γ_n, consisting of cyclic permutations of the variables x_i's for $1 \leq i \leq n$, and Δ_n, consisting of automorphisms isomorphic to the multiplicative group of roots of unity of order $s^n - 1$. In contrast, the isotropy group of the corresponding factorizable derivation is generated solely by Γ_n.

Finally, in the last chapter, we summarize the thesis and propose future work directions.