

Abstract:

This thesis studies the elliptic analogues of identities from combinatorics and hypergeometric series. Specifically, we extend the Fibonacci numbers, several other elementary identities, and summation and transformation formulas from hypergeometric series. We extend Fibonacci numbers with arbitrary weights and generalize a dozen Fibonacci identities. We propose an elliptic extension different from that given by Schlosser and Yoo. The proofs of most of the identities are combinatorial, extending the proofs given by Benjamin and Quinn, and in the q -case, by Garrett. Some identities are proved by telescoping. We provide elliptic extensions of elementary identities such as the sum of the first n odd or even numbers, the geometric sum, and the sum of the first n cubes. In the course of our study, we obtained an identity with many parameters, which appears to be new even in the q -case. In addition, we recover some q -identities due to Warnaar. We provide an alternate approach to obtaining expansion formulas on the lines of the well-poised Bailey lemma. We recover results due to Spiridonov and Warnaar and a new formula of this type. As a result, we are able to derive 19 transformation formulas for elliptic hypergeometric series. Among these transformation formulas, eight appear to be new even in the basic hypergeometric case, when $p = 0$.