Abstract

The goal of this thesis is to study the results on abstract module categories and Eilenberg-Moore Categories. This thesis is structured into two main parts. The first half focuses on the results on abstract module categories, beginning with a chapter that provides a brief survey of the literature on the subject.

In Chapter 2, we study local cohomology in the abstract module category, where we prove Grothendieck's vanishing theorem and Non-vanishing Theorem. In Chapter 3, we consider the categories of contramodule objects and comodule objects in a Grothendieck category over an entwining structure (A, C, ψ), where A is a k-algebra and C is a k-coalgebra. A measuring from an entwining structure (A', C', ψ') to (A, C, ψ) is considered, which induces pairs of adjoint functors between the categories of entwined contramodule objects as well as the categories of entwined comodule objects. We provide conditions under which these adjoint pairs are inverse equivalences. In this chapter, we also study separability, Frobenius, and Maschke-type results for functors between categories of entwined comodules and entwined contramodules.

The second half of the thesis focuses on the results on Eilenberg-Moore categories. We begin this part with a review of the literature on Eilenberg-Moore categories in Chapter 4. In Chapter 5, we consider representations of quiver taking values in the category of monads and comonads. We develop a categorical framework for modules and comodules over these representations. Our goal is to give conditions under which these categories become Grothendieck. We also study the categories of cartesian modules and comodules over these representations, which resemble quasi-coherent sheaves. We conclude this chapter by discussing the rational pairing between monad and comonad representations.

In Chapter 6, we consider a differential monad (U, θ, η, d) on a Grothendieck category, where $d : U \longrightarrow U$ is a derivation on U, and we define a d-derivation D on a module over the monad

U. We prove in this chapter that every hereditary differential torsion theory on the Eilenberg-Moore category of modules over a monad (U, θ, η, d) is differential. Further, we show that every d-derivation on a module in the Eilenberg-Moore category can be extended to the module of quotients.