# Abstract:

The current work develops mathematical models for the effect of complex surface topographies on electrically and mechanically driven flows using perturbation theory and eigenfunction expansions. The models developed have practical implications for the development of microfluidic devices for pumping, separation, mixing, and flow control and for the development of novel biomimetic surfaces such as lubricant-infused surfaces (LIS) and superhydrophobic surfaces (SHS). While the local features of the respective flow fields are resolved theoretically, significant attention is given to condensing the effects originating from the topography and other concomitant physical processes into proposed corrections to classically well-accepted paradigms for uniform surfaces, such as Helmholtz-Smoluchowski theory of electroosmosis, the Navier slip boundary condition, and the planarized interface. The analytical findings, when based on small parameters, are assessed for their usefulness against findings from full-scale numerical solutions. Every flow scenario of interest to this thesis can be split into transverse or parallel flow and longitudinal or perpendicular flow problems, depending on whether the driving force is directed parallel or perpendicular to the iso-contours of the surface topography. The complete anisotropic response to oblique driving forces, which can be the basis of novel separation strategies, can be inferred from the solution of these problems.

While the classical Helmholtz-Smoluchowski theory of electroosmosis was developed for planar and uniformly charged surfaces, the thesis attempts to generalize this theory through systematic and stepwise relaxation of these assumptions. For the perturbation procedures applied, the findings apply to spatially periodic systems and to small amplitude topographies. However, unlike past work, both the surface topography and surface charge variation (where present) can be specified as arbitrary functions.

Significant non-uniformities in surface charge may or may not accompany complexities of surface topography in electroosmotic flow. In parallel flow over uniformly charged surfaces, no far-field effect exists, though the flow field and volumetric charge density distribution inside the Debye layer are affected. In perpendicular flow over uniformly charged surfaces, electroosmotic flow (EOF) is always suppressed by the unevenness of the topography. The degree of EOF suppression in perpendicular flow has the interesting feature of being most significant at intermediate values of the Debye layer thickness to surface pitch ratio; indeed, the suppression is less significant in both highly concentrated and very dilute electrolytes. This non-monotonic behaviour is attributable to the tendency at intermediate wavelengths for the net volumetric charge density to suffer disproportionate attenuation in the troughs of the topography while coupling with the electric field strength, which is also lowered inside the troughs. The anisotropicity of the EOF in response to electric fields oriented obliquely to the topography is also characterized, based on which a novel strategy for separating nanoparticles is proposed.

Contrary to uniformly charged surfaces, the bulk fluid velocity is affected in both parallel and perpendicular flow when non-uniform surface charge densities accompany non-uniform surface charge topography. Also, the surface corrugations could resist or support the base flow

depending on the charge distribution. Based on this unique feature of non-uniformly charged surfaces, an approach to steer the flow direction just by manipulating the surface charge is suggested. Furthermore, it is noted that surface charge measurements may be contaminated by the charge variation effects, which need consideration.

The mechanically driven flows over complex surface topographies investigated in this work are inspired by LIS and SHS and consequently feature a surface-adherent lubricant layer. The resultant two-phase flow problem is resolved analytically and given a global characterization through the effective slip length formalism that requires the prescription of a Navier slip boundary condition. First, the slip length for rectangular topographies filled with lubricants up to the peaks of corrugations is determined using an eigenfunction expansion-based approach free of perturbation parameters, thus avoiding restrictions on the height of the topographic features. Then, for arbitrary-shaped topographies overfilled with lubricants, the slip length is obtained for parallel flows where the interface is undisturbed. Key topographic parameters such as lubricant fraction, topography amplitude, and interface height relative to the topography peaks were studied for their effects on slip length. Additionally, the effect of the ratio of viscosities of the primary fluid and the lubricant was examined. Our findings indicate that the viscosity ratio and lubricant fraction on the plane passing through the topography peaks are more critical in determining slip length than the topography height itself. Furthermore, surfaces with different topography shapes that entrap the same lubricant volume and fraction exhibit varying slip lengths, underscoring the significance of optimizing topography shapes for enhanced performance.

Finally, an analytical expression is presented to predict the deformation of the fluid-fluid interface over an LIS or SHS in transverse flow based on a perturbation procedure. Interface deformation, driven by flow over corrugated surfaces, can lead to lubricant depletion. Surface tension is identified as the key factor resisting deformation, with moderate capillary numbers posing the greatest risk for lubricant loss. This analysis provides a comprehensive understanding of the factors governing fluid flow over LIS and highlights the importance of surface tension, viscosity, and topography optimization in controlling interface deformation and slip characteristics.