Abstract of Ph.D. Thesis
"Online Algorithms for Geometric Problems"
Mr. Satyam Singh (2019MAZ8758), Research Scholar

Abstract

The online model of computation has gained significant interest for more than five decades due to its ability to effectively capture real-world phenomena like the irreversibility of time (decisions) and the unpredictability of the future (input). In this thesis, we study three fundamental graph-theoretic problems: maximum independent set (MIS) problem, minimum dominating set (MDS) problem and its variants, and minimum coloring (MC) problem and two set-theoretic problems: hitting set problem and piercing set problem in the online setup.

The graph-theoretic problems considered here, apart from the minimum connected dominating set (MCDS) problem, are studied in the classical online model. For the online MIS and MDS problems, in the classical online setup, the disastrous $\Omega(n)$ lower bounds on the competitive ratios are known for general graphs over two decades, which also holds for interval graphs, where n is the number of vertices. In this thesis, for the online MIS, MDS and maximum independent dominating set (MIDS) problems, we demonstrate that for a graph with an independent kissing number at most ζ, there exist well-known greedy algorithms that achieve optimal competitive ratios of ζ. In contrast, for the online MCDS problem, we show that there exists a deterministic algorithm having an optimal (asymptotic) competitive ratio of $2(\zeta - 1)$. For the online MC problem, we show that there exists an algorithm that achieves a competitive ratio of at most $\zeta'(\lfloor \log M \rfloor + 1)$, which is an improvement over the existing best-known result of ζ. Here, ζ' and ζ are independent kissing numbers of similarly sized fat objects having widths in the range $[1, 2]$ and $[1, M]$, respectively. The results obtained in this thesis for these problems heavily depend on a graph parameter: independent kissing number. Thus, we investigated the value of the independent kissing number for various families of geometric objects.

Then, we switch our attention to set-theoretic problems in the online setup. It is known that no online algorithm can obtain a competitive ratio better than $\Omega(\log M)$ for hitting M intervals in the range $[1, M]$ using points $\mathbb{Z}$. Due to this pessimistic result, we consider the online hitting set problem for a finite family of translates of an object in $\mathbb{R}^d$ using points in $\mathbb{Z}^d$. For the low dimensional unit balls and hypercubes in $\mathbb{R}^d$ $d \leq 3$), we present deterministic algorithms having almost tight constant competitive ratios. For the unit hypercubes in $\mathbb{R}^d$ ($d \geq 3$), we present an $O(d^2)$-competitive randomized algorithm, while for unit balls in $\mathbb{R}^d$, we present an $O(d^4)$-competitive deterministic algorithm. Additionally, we investigated the lower bounds of the hitting set problem for unit balls in $\mathbb{R}^d$ (for $d \leq 3$) and unit hypercubes in $\mathbb{R}^d$ (for $d \in \mathbb{Z}^+$), and we show that every deterministic algorithm has a competitive ratio of at least $d + 1$. We prove that the competitive ratio of every deterministic online algorithm for piercing intervals in $\mathbb{R}$ is at least $\Omega(n)$. Due to this hopeless lower bound, in this thesis, we consider the piercing set problem for similarly sized fat objects in $\mathbb{R}^d$. We propose a deterministic online algorithm for similarly sized fat objects in $\mathbb{R}^d$. For homothetic hypercubes in $\mathbb{R}^d$ with side length in the range $[1, M]$, we propose a $(3^d\lceil \log_2 M \rceil + 2^d)$-competitive deterministic algorithm. Furthermore, we obtain a deterministic lower bound of the problem for homothetic hypercubes in $\mathbb{R}^d$. Note that piercing translated copies of a convex object is equivalent to the unit covering problem, which is well-studied in the online setup. Our result yields an upper bound of the competitive ratio for the unit covering problem when the corresponding object is any convex object in $\mathbb{R}^d$.