Abstract

Over the past few decades, the study of flexural wave propagation through metamaterials has garnered significant attention due to their remarkable wave manipulation capabilities. Depending on the characteristics of the representative unit cell, metamaterials can either facilitate or inhibit wave propagation within certain frequency bands, making them highly applicable in fields such as vibration isolation, energy harvesting, wave filtering, and sound insulation. Despite these advances, substantial research opportunities remain in the analytical solutions of wave dispersion in metamaterials with various mechanisms and configurations.

This study presents analytical solutions for wave dispersion characteristics in one-dimensional metamaterial systems, ranging from simple to complex configurations, including discrete monocoupled systems, discrete multicoupled systems, continuous multicoupled systems, and continuous nonlinear systems.

First, a generalized monocoupled system is introduced, combining elements with inertial amplification, negative mass, and negative stiffness. A hierarchical stiffness system is conceptualized to produce multiple attenuation peaks. A general framework employing rational polynomials is developed, from which closed-form expressions for the positions of attenuation peaks and bounding frequencies of propagation bands can be derived for any undamped monocoupled system.

The study also explores wave dispersion in a discrete bicoupled system formed by coupling two spring-mass chains, analyzed for both straight and zigzag couplings. The analytical dispersion relation is derived using the dynamic stiffness matrix and the Bloch-Floquet theorem, with invariants characterizing the band structure. Mechanisms of band veering and locking are examined, identifying a maxon at wave locking and a roton at unlocking in the zigzag coupling. Sensitivity analysis on the invariant plane reveals how various parameters influence wave propagation characteristics.

For continuous systems, the study investigates flexural wave propagation through a rigid elastic combined metabeam (RECM) considering the finite dimensions of the rigid mass. The rotary inertia of the rigid body introduces a local resonance (LR) band, which can bridge the gap between two Bragg scattering (BS) bands, resulting in an ultra-wide stop band for specific combinations of governing non-dimensional parameters. Additionally, a vibration isolator device based on the rigid elastic combined beam is developed, exhibiting a wide bandgap due to double antiresonance peaks, with experimental validation of the isolator conducted. A parametric study varying the geometric properties of the rigid elastic isolator system enhances the understanding of bandgap and

attenuation characteristics within the attenuation band. Furthermore, wave dispersion in a chain of masses connected with a designed rigid elastic isolator is studied and validated, with analytical solutions obtained through the transmittance study of the representative unit cell.

Lastly, for continuous nonlinear systems, the amplitude-dependent dispersion relation of a damped beam supported by nonlinear springs is studied, considering both softening and hardening cubic nonlinearities along with viscous and strain rate damping. Using a multiscale method, the wave dispersion equation is analytically derived and validated against numerical results. The research provides closed-form equations for frequency shifts in undamped systems due to nonlinearities and time-dependent dispersion in damped systems.